**PROJECT TWO: MAKING INVESTMENT DECISIONS BASED ON DATA**

*One important application of statistics is providing probabilistic data analyses in the face of uncertainty. In this assignment, you have the opportunity to earn points by making investment decisions that maximize return while minimizing risk, based on assumed situations.*

**Background.** The “institutional leveraged loan” market has a value close to one trillion dollars. Leveraged loans have a risk of defaulting, meaning that the lenders do not get paid; but “insurance” can be bought on loans, reducing the impact if a borrower defaults.

(For examples of reporting on “institutional leveraged loans”, see <http://247wallst.com/banking-finance/2015/12/18/leveraged-loan-default-rate-forecast-at-2-5-for-2016/> )

You are an analyst at “Obtain Wise Zen” (OWZ), a multi-billion dollar investment firm. You have been assigned to examine the market opportunities in a set of 121 loans known collectively as the “Dicing Annex” (DA). There are opportunities to sell and to buy insurance for the loans within DA, but perhaps it may make more sense to walk away.

Selling insurance to DA

The lenders for DA are seeking insurance to guard against possible losses. Analysts at OWZ believe that the chance of any loan defaulting over the course of a year is 2.5%. OWZ currently does not have any stake in any of the 121 loans, but OWZ has been offered the opportunity to sell insurance to the lenders, as follows:

* OWZ promises to insure at most **n** of the 121 loans (where **n** is between 1 and 121)
* OWZ will be paid $4,000,000 (4M) for each loan thus insured
* For the first **n** defaults within DA within the year, OWZ will pay $170,000,000 (170M) per default

This deal is known as the **Insure(n)** option, where **n** is the number of defaults insured. For example, OWZ could choose to insure **n** = 2 loans. In that case, OWZ would be paid $8M at the start; if there are no bankruptcies, then OWZ’s net income from the deal is $8M. If there were one bankruptcy during the year, OWZ’s net earnings would be +8M – 170M = –162M. If there were two bankruptcies during the year, OWZ’s net earnings would be +8M – 340M = –332M. If there were three or more bankruptcies during the year, OWZ’s net earnings would still only be –332M, because in this example OWZ only agreed to insure the first **n** = 2 loans.

OWZ can also borrow money, at an annual interest rate of 7%; so if $100 is borrowed, a total of $107 must be paid back.

Buying insurance on possible DA defaults

A large bank (“Shallow Inked Lion”, or SIL) is also selling insurance on DA; OWZ thus has the opportunity to buy insurance on the loans within DA. SIL has three insurance packages on DA:

* Tier A: the buyer gives $480,000,000 (480M) to SIL. In return, SIL will give the buyer $230,000,000 (230M) for each default, up to three defaults (for a maximum possible payout to the buyer of $690 million)

* Tier B: the buyer gives $480,000,000 (480M) to SIL. In return, SIL will give the buyer $800,000,000 (800M) for each default after the third, up to five defaults (so defaults #4 through #8 are insured, with a maximum possible payout to the buyer of $4 billion)
* Tier C: the buyer gives $480,000,000 (480M) to SIL. In return, SIL will give the buyer $8,000,000,000 (8000M) for each default after the eighth (so defaults #9 through #121 are insured, with a maximum possible payout to the buyer of $904 billion)

These are three separate, non-overlapping packages; one could, in principle, buy all three (at a cost of $1.44 billion), and thus receive a payout no matter how many defaults there were.

Other investment options

OWZ can buy risk-free investments; risk-free investments pay 2.14% annually (the rate of 10-year Treasury bonds).

You have been asked to make an investment based on the available information.

**Part one. Making investment options given a single scenario**

You have been asked to provide summary statistics for the various investment options, assuming that the rate of defaults is **p** = 0.025. In particular, the senior partners want you to examine five investment possibilities:

* Insure(**n** = 60) loans
* Insure(**n** = 120) loans
* Borrow $480M and buy Tier A insurance from SIL
* Borrow $480M and buy Tier B insurance from SIL
* Borrow $480M and buy Tier C insurance from SIL

For each of the five investment possibilities, the partners want to know the following values:

* **Net expected value** of the investment after payouts, interest payments, etc.
* **Standard error** of the investment
* **Probability** that the investment will result in a net positive income
* **Net expected downside risk** of the investment (the expected value of possible losses, treating any positive values as zero.)

For example, if an investment had four equally likely outcomes of +1, +2, +3, and –6, the expected value would be zero, the standard error would be 3.5355, and probability of a positive value would be 75%, and the expected downside risk would be –1.5 (= 0\*0.25 + 0\*0.25 + 0\*0.25 – 6\*0.25).

1. [2 points] Fill in the summary table with the corresponding expected values, standard errors, probabilities, and expected downside risks.
2. [1 point] A staffer recommends suggests that combinations of investments could be profitable, under certain circumstances. For example, if OWZ were to **sell** n=120 policies (raising $480M) and then turn around and **buy** Tier A (at $480M), and if there are only 3 defaults, then OWZ ends up with (480M – 480M – 3\*170 + 3\*230) = 180M. Of course, combinations are only profitable under specific circumstances. What is P(exactly 3 defaults) if **p** = 0.025?
3. [3 points] There is an internal debate within OWZ as to how to invest money. One staffer recommends “Insure(120), Buy A”; another staffer recommends only investing in investments that have a positive expected value; a third staffer recommends minimizing expected downside risk; etc. What investment decision would you recommend based on the assumption that **p** = 0.025? Report your investment decision based on answering the following questions. Your grade will be based on the expected value of your investment choices:
   1. Do you wish to sell insurance for defaults within DA? If so, how many defaults are you willing to insure (being paid $4M x **n** for insuring the first **n** loans against default, but possibly paying $170M per default for the first **n** defaults)?
   2. ~~Do you wish to borrow any money (at an interest rate of 7%)?~~ If you have not raised sufficient funds in part a. to cover the total expenses in parts c., d., e., and f., the balance due will automatically be borrowed at an interest rate of 7%.
   3. Do you wish buy Tier A insurance (by paying $480M to SIL)?
   4. Do you wish buy Tier B insurance (by paying $480M to SIL)?
   5. Do you wish buy Tier C insurance (by paying $480M to SIL)?
   6. How much do you wish to invest in 10-year Treasury bonds?

**Part two. Making investment options given multiple possible scenarios**

Assume that the probability of **p** = 0.025 is just the “best guess” as to the default rate. In fact, analysts at OWZ say that there is a 40% chance that **p** = 0.025, a 30% chance that **p** = 0.015, and a 30% chance that **p** = 0.035. (Note that (0.4 x 0.025) + (0.3 x 0.015) + (0.3 x 0.035) = 0.025, so the analysts felt that if one number had to be used, 2.5% is appropriate, since 2.5% is both the expected value and the most likely of the three outcomes.)

Before making the investment decision, you have the opportunity to pay $80,000,000 (80M) to know whether the probability will be 0.025, 0.015, or 0.035, and adjust your investment strategy accordingly. Some at OWZ think it would be valuable to know the probability in advance; others say that the cost of getting that information is too high.

1. [3 points] What is the maximum amount you should be willing to spend to know the probability (i.e., what is the information worth)?
2. [3 points] You have an opportunity to make an investment decision; as before, your grade will be based on the expected value of your investment choices, this time taking into account that **p** may vary. Answer the following questions:
   1. Do you wish to spend $80M to know the value of **p** in advance?
   2. If the answer to **a** is yes, then answer the following three questions (and skip **c**); if the answer to **a** is no, then skip this question and answer question **c**.
      1. If **p** = 0.015, how many defaults do you wish to insure, how much money do you wish to borrow, and which (if any) of Tier A, Tier B, and Tier C do you wish to purchase? How much do you wish to invest in 10-year Treasury bonds?
      2. If **p** = 0.025, how many defaults do you wish to insure, how much money do you wish to borrow, and which (if any) of Tier A, Tier B, and Tier C do you wish to purchase? How much do you wish to invest in 10-year Treasury bonds?
      3. If **p** = 0.035, how many defaults do you wish to insure, how much money do you wish to borrow, and which (if any) of Tier A, Tier B, and Tier C do you wish to purchase? How much do you wish to invest in 10-year Treasury bonds?
   3. If the answer to **a** is no, then answer the following: regardless of what **p** is, how many defaults do you wish to insure, how much money do you wish to borrow, and which (if any) of Tier A, Tier B, and Tier C do you wish to purchase? How much do you wish to invest in 10-year Treasury bonds?

**Part three. More advanced scenarios**

1. [2 points] Suppose that the $80M does not guarantee perfect information. Instead, suppose that spending $80M results in the following  
   * If **p** really is 0.025, there is an 80% chance that **p** will be estimated as 0.025, a 10% chance that **p** will be estimated as 0.015, and a 10% chance that **p** will be estimated as 0.035
   * If **p** really is 0.015, there is a 90% chance that **p** will be estimated as 0.015, and a 10% chance that **p** will be estimated as 0.025
   * If **p** really is 0.035, there is a 90% chance that **p** will be estimated as 0.035, a 10% chance that **p** will be estimated as 0.025

How much less is the (possibly inaccurate) estimate about **p** worth now, compared to your answer for **4** (where **p** could be known with 100% accuracy)? Express your answer as the difference between the amount you should be willing to pay in **Q4** versus the amount you should be willing to pay now.

1. [1 point] One assumption throughout the analysis is that all loans are at equal risk of default. In fact, some loans are riskier than others. Suppose that 24 of the 121 loans are “energy sector” loans with a default probability of 8.6%, and the 97 loans are “non-energy sector” loans with a default probability of 0.99%. Given these assumptions, what is P(exactly 3 defaults)?